Adaptive Voronoi Binning in Muon Radiography for Detecting Subsurface Cavities

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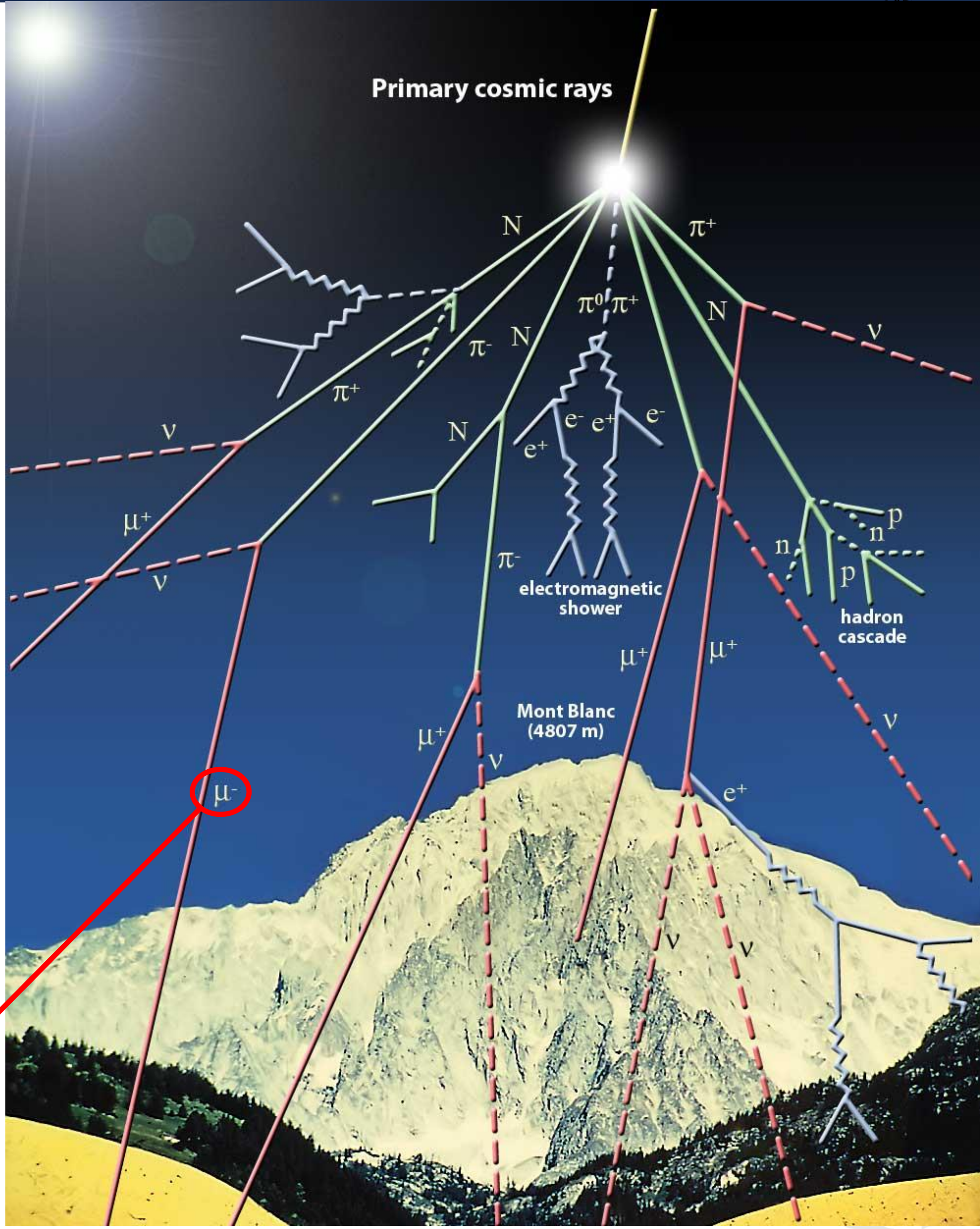
Cosmic Rays

- Cosmic rays are divided into primary and secondary:
  - The primaries impact the atmosphere from space and originate mainly from outside the solar system:
    - 98% nuclei:
      - 87% p
      - 12% He
      - 1% other nuclei
    - 2% e^+ and e^-
  - Secondaries are produced by the interaction of primaries with nuclei in the atmosphere:
    - X-ray, neutrons, mesons (such as pions and kaons), electrons and muons.



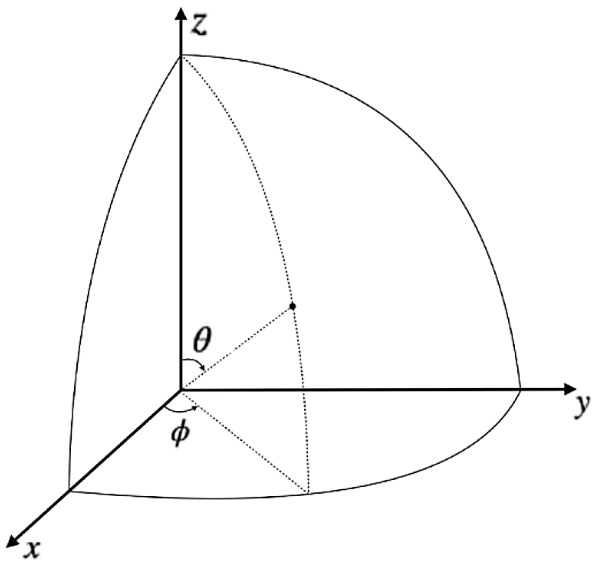
- $m_\mu \sim 105\mathrm{MeV}/c^2 \sim 200\ m_e$
- High penetrating power in the mater
- Most abuntant particles on the ground
- Life Time $\tau_\mu = 2.2\ \mu s$

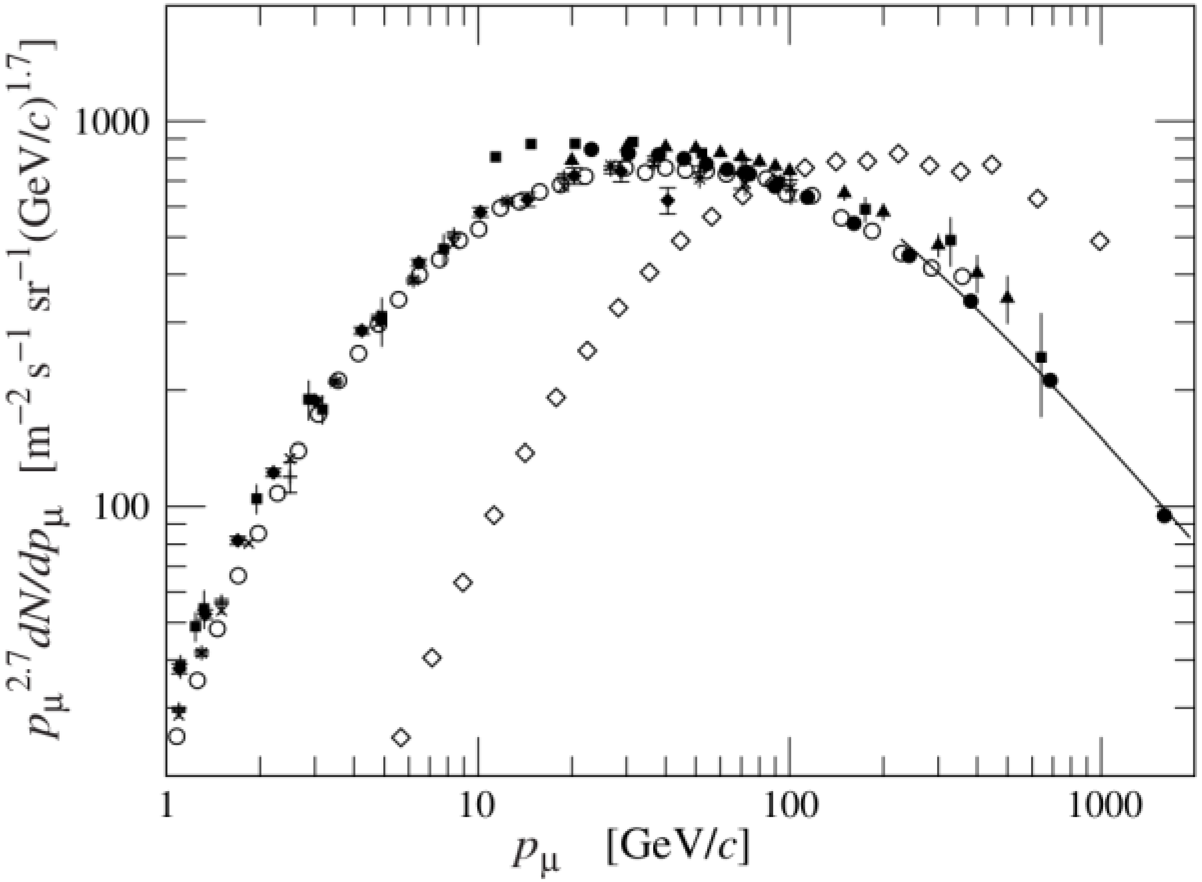
Atmospheric Muons

- The Muon Flux at sea level in vertical it is approx $\Phi = 1/(cm^2 s)$
  - But this depends on the zenith angle and its energy

- The flow decreases with the increase of θ as $cos^2\theta$

- θ: Zenith angle
- φ: Azimuth angle
- $90 - \theta$: Elevation





The spectrum of muons at sea level for θ = 75 is indicated by the empty diamonds while the other markers refer to the muon spectrum for θ = 0. The average energy of muons increases as the zenith angle of origin increases. The ordinate is multiplied by $p_\mu^{2,7}$ to compress the spectra.

Muon-matter interaction

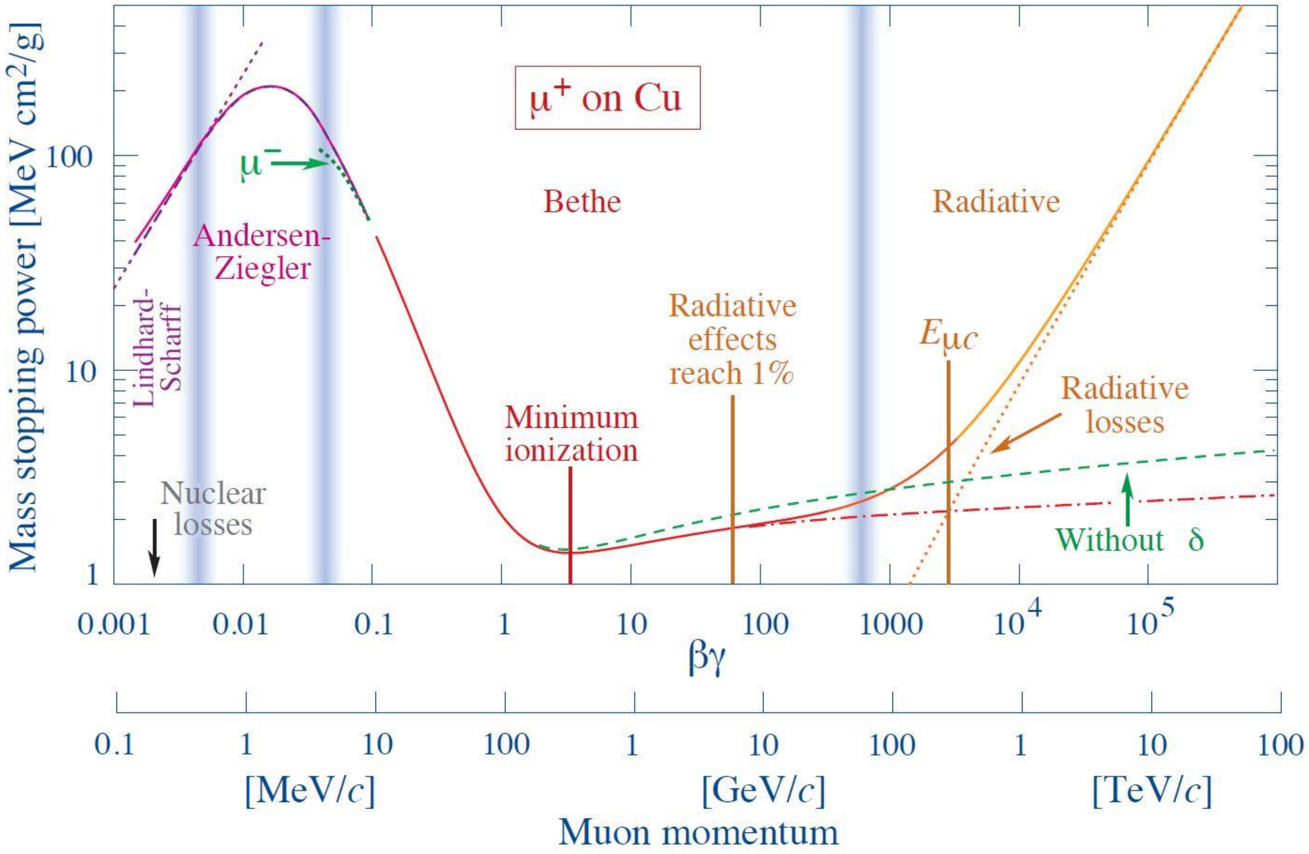
- The interaction of muons with matter is described by processes which are dominant at different energies:
  - Atomic excitation and ionization (below 100GeV)
  - Radiative processes (over 100GeV)

- The quantity that describes the energy loss of muons when they interact with matter is the Stopping Power:

$$-\left(\frac{dE}{dX}\right) = a(E) + b(E)E$$

- $a(E)$ describes the energy loss upon excitation and ionization and is described by the Bethe-Block formula.

- $b(E)E$ describes the loss of muon energy due to radiative effects, most of the muons involved in muon radiography are below 100 GeV.



Stopping power of muons in copper as a function of the kinetic energy.
Our energy range of interest is between Minimun Ionizatiom and 100GeV.

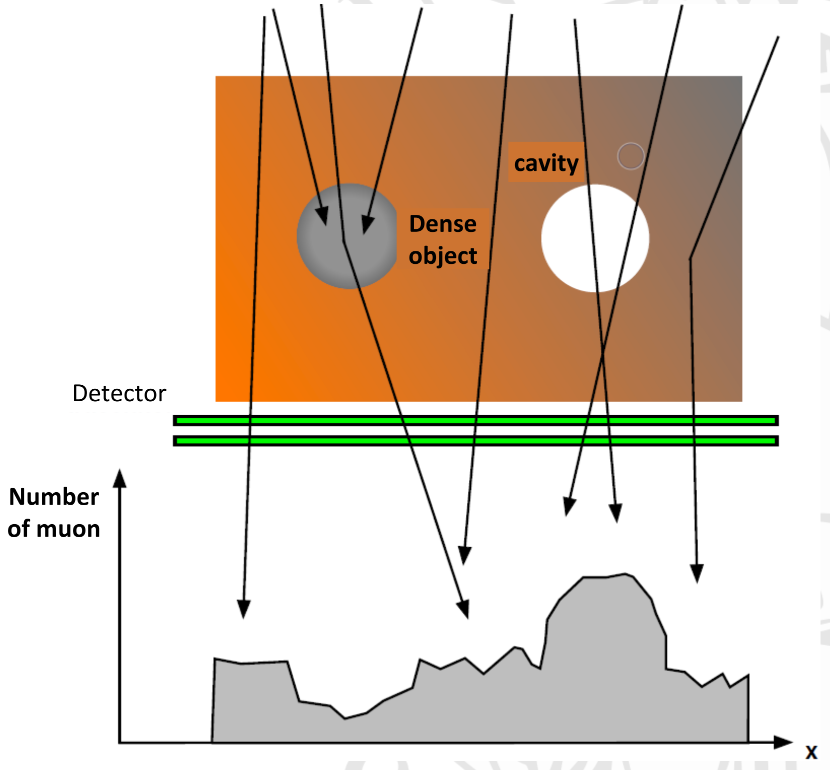
Muon Radiography

- The muon radiography technique exploits the penetration capacity of muons present in cosmic rays to make radiography of very large targets.

- This technique bases its operation on the energy loss of muons when they pass through a dense body.

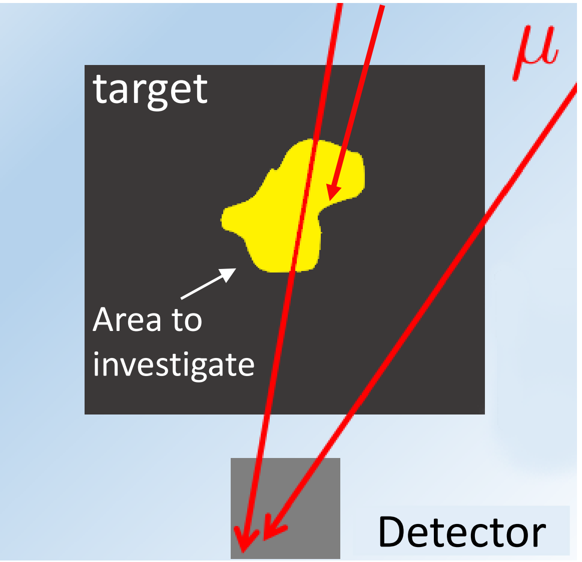
- Since muons are less likely to interact, stop and decay in low density matter than in high density matter, a larger number of muons will travel through the low density regions of target objects in comparison to higher density regions.

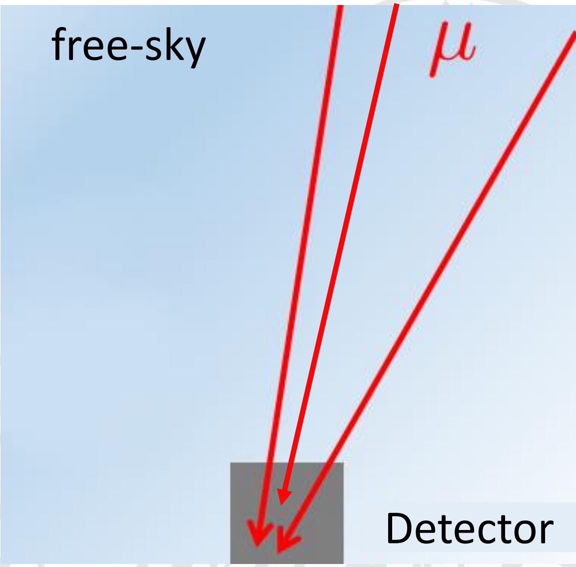
- Muography is a non-invasive technique since it exploits natural radiation present on the entire surface of the earth and we are continuously traversed by these particles.



Muon Transmition technique

1. Observing the number of counts in the presence of the target ($N_{target}(\theta,\varphi)$).
2. Observing the number of counts without the target ($N_{target}(\theta,\varphi)$), also called Free-Sky configuration.
3. Compare the measured transmission with that expected at a fixed density.





- A muon with a given momentum p_{min}, before being stopped, will be able to cross on average a certain thickness of material to which we can associate an opacity *X*.

- Fixed a certain angular direction (θ,φ) and a certain opacity, the expected value of the flux transmitted through the target can be estimated as the integral from p_{min} to infinity of the differential flux in that direction $\Phi_{trasmitted} = \int_{p_{min}}^{\infty} d\Phi(\theta,\varphi)\, dp$

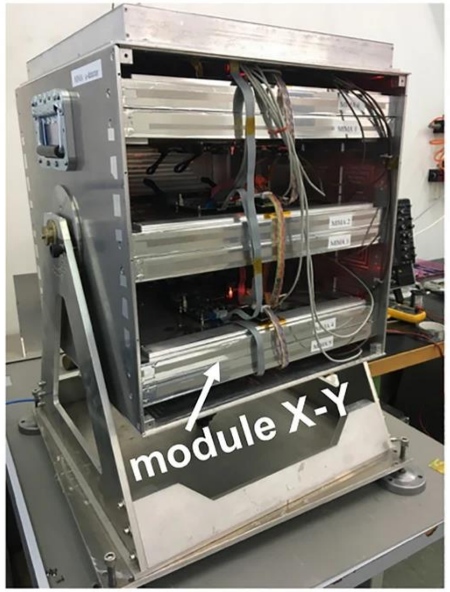
$$T_{measured} = \frac{N_{target}(\theta,\varphi)}{N_{free-sky}(\theta,\varphi)} \cdot \frac{t_{free-sky}}{t_{target}}$$

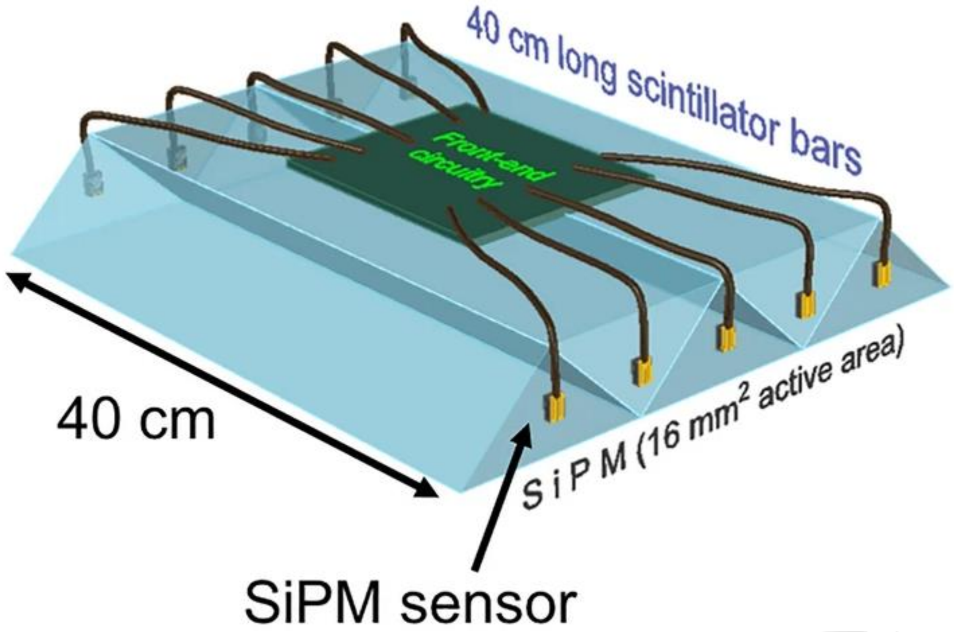
$$T_{relative} = \frac{T_{measured}}{T_{expected}}$$

- > 1 Cavity?
- <1 Dense Object?
- =1 Match with simulation

MIMA

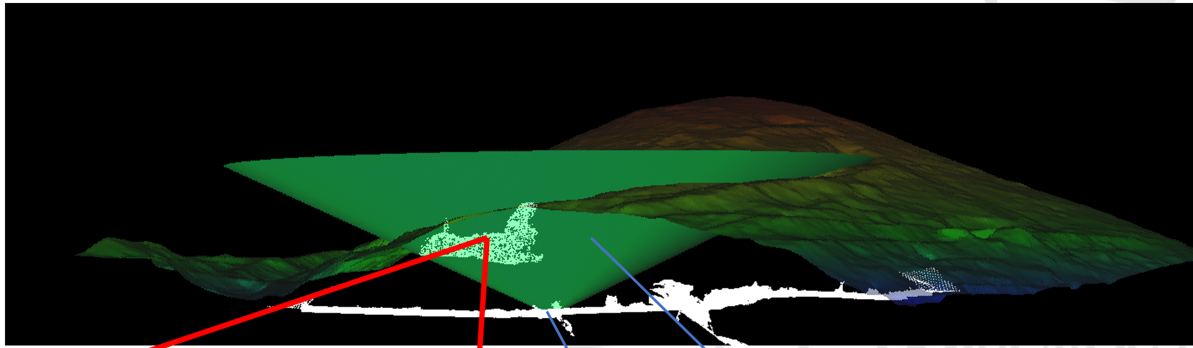
- MIMA (Muon Imaging for Mining and Archaeology)
- Field: geology, archaeology and civil engineering.
- Aluminium cover: (50 x 50 x 50) cm^3
- Altazimuth orientation.
- Six tracking planes organized in orthogonal pairs, forming three compact X-Y modules.
- Active surface area of each plane: (40 x 40) cm^2
- Resolution: 7 (mrad) $\sim 0.40\ Deg$
- Acceptance: 60 degrees.





Temperino

- MIMA detector was positioned inside the mine with a vertical orientation, namely at $\theta = 0^{\circ}$
- Reference density with which the expected transmission was calculated: $2.65\ g/cm^3$
- Brighter areas = Higher relative transmission: region with a lower density than expected.
- Darker areas = Lower relative transmission: regions with a higher density than expected.



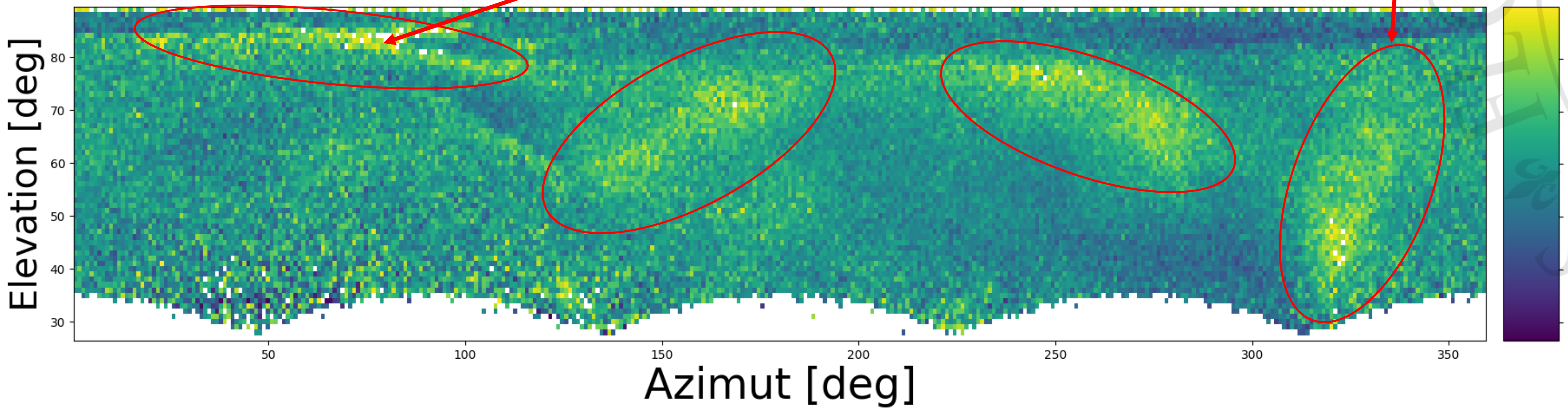
Acceptance Cone

MIMA Measuring Point

Cavity

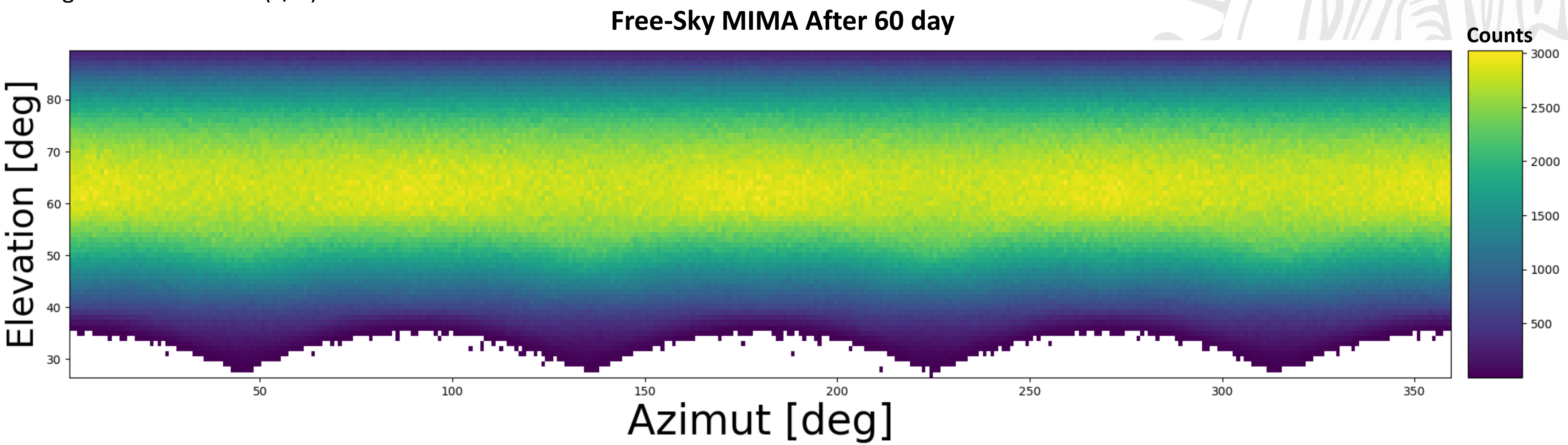
Cavity

Relative Transmission



Why use variable binning

- When collecting particle counts, as done with the MIMA detector, they are influenced both by the geometry of the detector and the flux variation with the azimuth angle.
- The counts will be higher at the center of the detector and decrease towards the edges, .
- For this reason, using an adaptive binning technique would allow each bin to have an S/N value like all the others. The adaptive binning that we propose to use is the Voronoi Tassellation.
- By utilizing Voronoi binning, the size and shape of each bin are automatically determined based on the distribution of data points and the local signal-to-noise ratio (S/N).

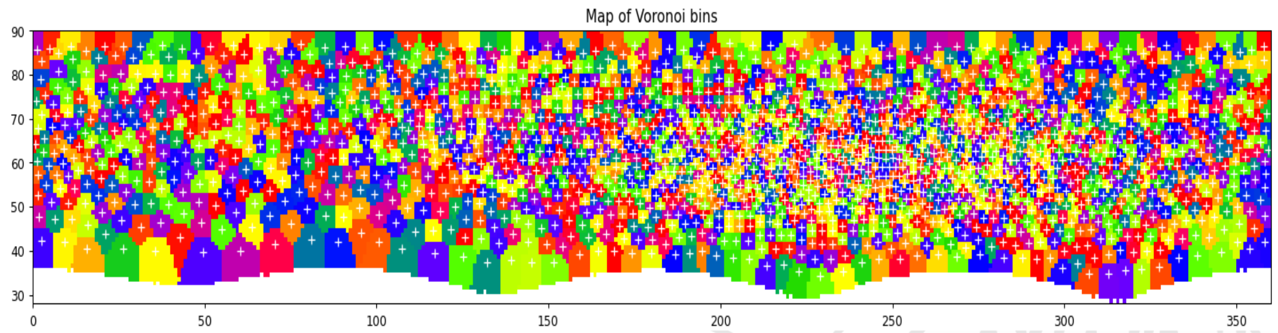


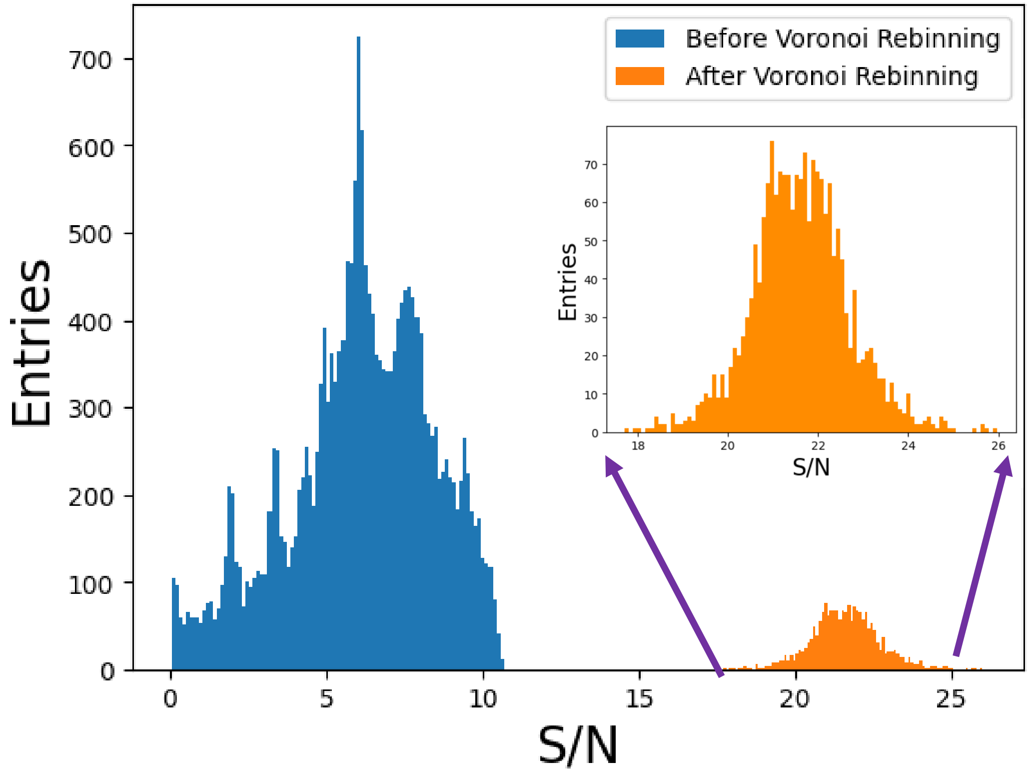
Result

- By utilizing Voronoi binning, the size and shape of each bin are automatically determined based on the distribution of data points and the local signal-to-noise ratio (S/N).

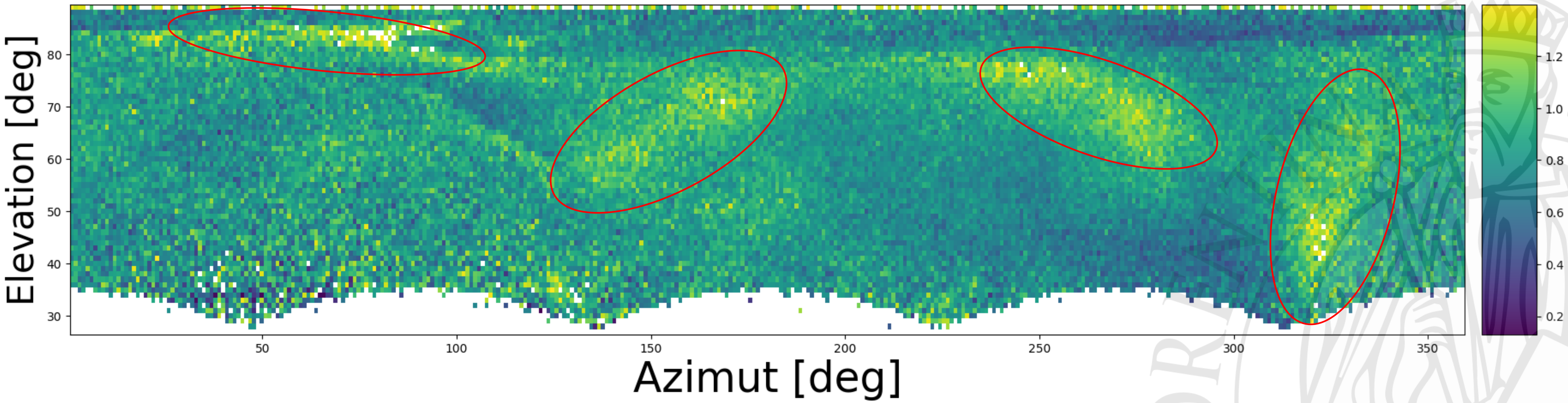
- We started with the expected counts in the target configuration and chose a signal-to-noise (S/N) threshold equal to twice the square root of the maximum cont value $(S/N)_T = 2 \cdot \sqrt{N_{max}}$.

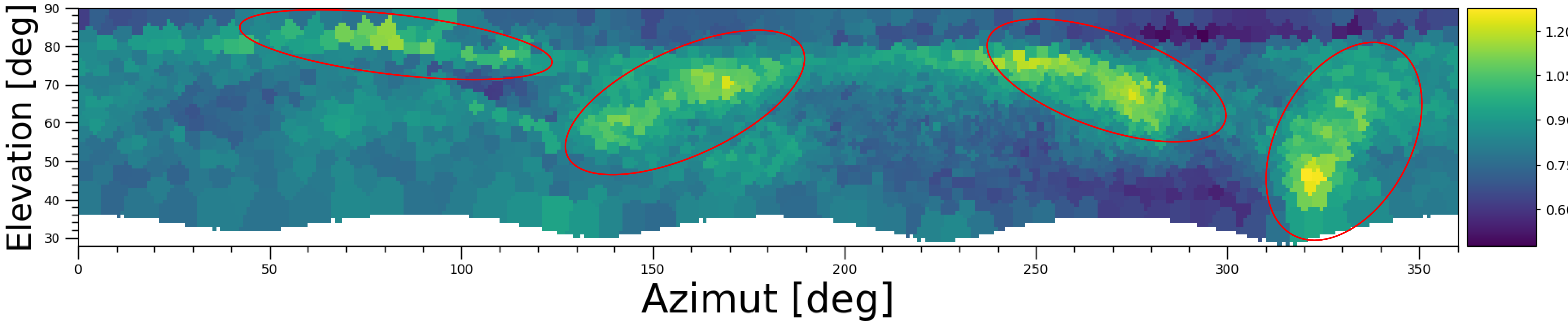
- This choice allowed us to transist from an average S/N of 8 to an S/N of 22 in the aspected trasmition counts.



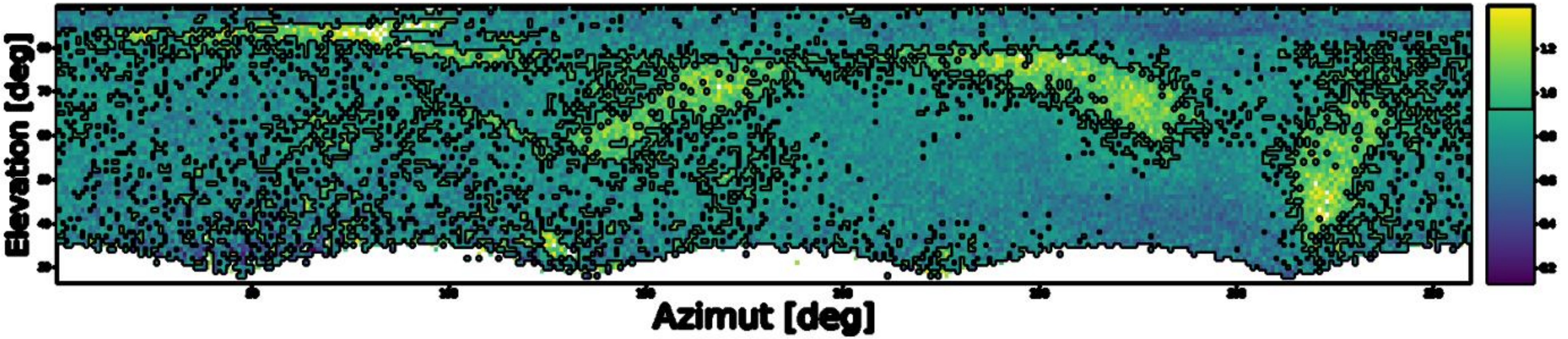


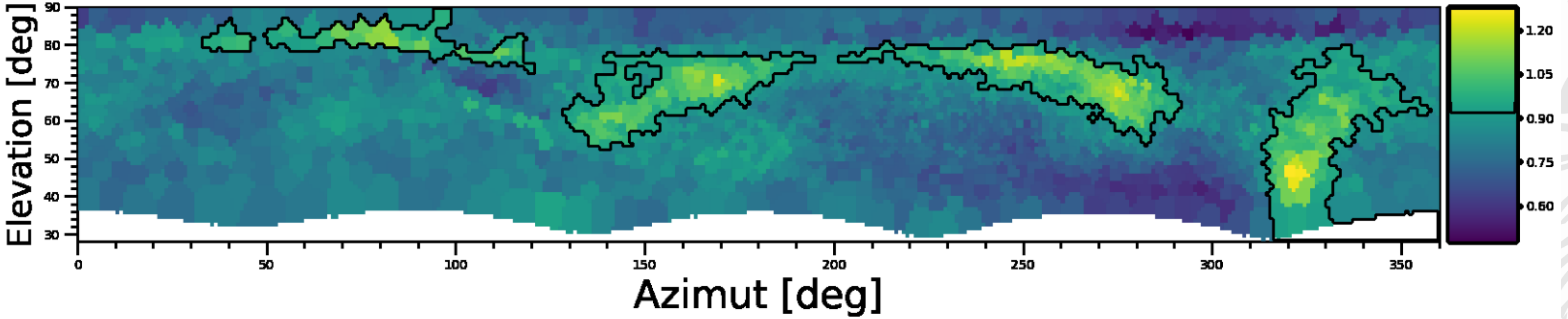
Relative Transmission before and After Voronoi Tassellation





Future work: Automatic Draw Contours and ML





The plot of the relative trasmition after the Voronoi tassellation shows how the use of that technique can be accompanied by a procedure for automatically identifying the presence of cavities. In fact, since there is a greater contrast, by applying an algorithm for the automatic drawing of the anomalies, the noise decreases significantly.

Conclusion and Future Work

- In this study, the aim was to introduce muonic radiography and its application in the search for cavities in the Temperino mine, and how adaptive binning can be improve its performance.
- The study demonstrated how applying the Voronoi procedure to the relative transmission image improved visibility, particularly in areas with higher transparency, which corresponded to cavities.
- The performance of muonic radiography, coupled with adaptive binning, highlights its potential as a powerful tool for studying complex systems and detecting underground structures. The technique’s ability to reveal detailed information about the interior of materials and structures makes it an asset in various scientific and engineering applications.

BACK-UP

Voronoi Procedure

1. Calculate the local S/N ratio for data point. This can be done by considering the signal and noise levels within each bin, In our case the signal $\mathrm{S} = N_i$ is the number of counts while the noise is its square root $\mathrm{N} = \sqrt{N_i}$.

2. Determine the desired level of adaptivity for Voronoi binning. This may involve setting thresholds or criteria based on the S/N ratio. In our case we want each time a bin is merged that the new one has a fraction of the S/N equal to 80% of the default one, this in turn was obtained as 2 times the square root of the maximum count of the data point $(S/N)_T = 2 \cdot \sqrt{N_{max}}$.

3. Identify bins that meet the adaptivity criteria. These are typically bins with a low S/N ratio, indicating regions where larger bin sizes are required.

4. Apply Voronoi tessellation to the selected bins. Voronoi tessellation calculates the Voronoi cells based on the centroids of the selected bins, generating adaptive bin sizes and shapes.

5. Recalculate the S/N ratio for the newly formed Voronoi bins. This step ensures that the binning remains adaptive and accurately reflects the local S/N characteristics of the detector.

6. Repeat the process iteratively for each data point, merging the initial bins that didn't meet the criteria with the closest Voronoi bins created.

Muon and Relativity

The muon velocity is very high (it's about $v = 0.9992 \times c$ where c is the speed of light $c = 3 \times 10^8 m/s$)

If we apply the classic kinematics laws:

- The distance traveled s during their lifetime τ_μ would be $s = v \times \tau_\mu \approx 660m$, in the atmosphere.
- How do they get to earth by traveling 15 km of atmosphere if they decay within 660m of their formation?

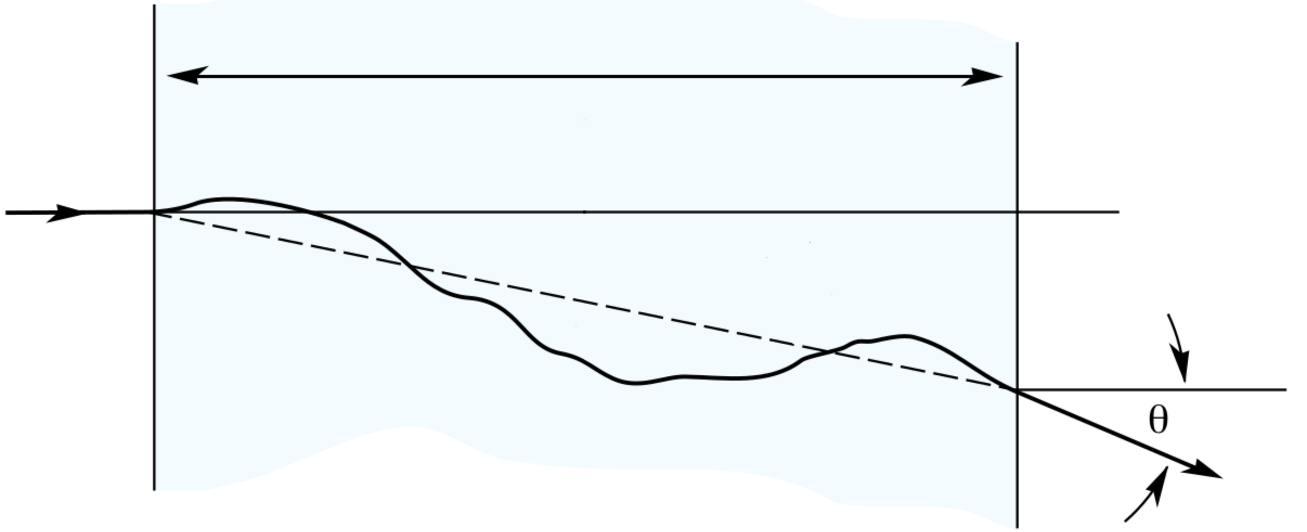
If we consider the relativistic kinematics:

- For a relativistic particle ($v \approx c$) the Lorentz factor is defined as $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ and in this case it's $\gamma \approx 25$
- The lifetime of the muon is $\tau_\mu = 2.2\ \mu s$ when measured in the muon reference frame.
- But if we measure the same phenomenon in the reference system of an observer on the ground (like us), we have the so-called time and length dilation phenomenon: $\boldsymbol{\tau'_\mu = \gamma \times \tau_\mu = 55\mu s}$ therefore $s' = v \times \tau'_\mu \approx$ 16Km

Muon-matter interaction

- A charged particle when interacting with a medium can be deflected by a certain angle as a result of numerous scatterings.

- If the overall deflection of the particle's trajectory is small, it is distributed according to a Gaussian.



Trajectory deflection due to EM interactions with the e^- of nuclei and with the nuclei themselves of the material:
θ depends on $\boldsymbol{X_0}(Z)$
θ depends on the momentum of the particle

$$\sigma(\theta) = \frac{13.6\ MeV/c}{p}\sqrt{\frac{x}{X_0}}$$

- x thickness of the material traversed
- p momentum of the particle
- X_0 radiation length